Single-world exchangeability conditions for a large class of regimes

Catharina Stoltenberg
EuroCIM
April 19, 2026

University of Oslo

Ongoing work

This presentation is based on ongoing work with

Thomas Richardson, James Robins, and Mats Stensrud.

What we do

Present new exchangeability conditions that are

- General: cover a large class of treatment regimes
- Unified: reduce to well-known conditions in standard settings
- Minimal: implied by existing conditions

and regime-specific SWIGs that can incorporate regime-specific information.

A simple setup

Variables

Factual variables:

- $A \in \mathcal{A}$: observed treatment
- $Y \in \mathcal{Y}$: observed outcome
- U: unobserved variable

A regime g assigns treatment $A^+(g) = g(A)$ for some function $g : \mathcal{A} \to \mathcal{A}$.

- Two distinct treatments variables: natural A and regime-assigned $A^+(g)$
- Regime g is a dynamic natural treatment value (NTV) regime
- Special case: regime g is a static regime if $g(a) = \tilde{a}$ for some $\tilde{a} \in \mathcal{A}$ and all $a \in \mathcal{A}$.

$Y(g) \equiv Y(A^+(g))$: the potential outcome under regime g.

Basic assumptions

We assume consistency,

$$Y = Y(A) \quad \text{a.s.},$$

and positivity,

$$P(A = a) > 0 \Rightarrow P(A = g(a)) > 0 \quad \forall\, a \in \mathcal{A}.$$

The estimand of interest is

$$E[\,Y(g)].$$

Exchangeability conditions

A new exchangeability condition

Define

$$S_g(a) = \{g(a)\} \cup \{a' \in \mathcal{A} : g(a') = g(a)\} \qquad \forall\, a \in \mathcal{A}$$

The new exchangeability condition is

$$Y\big(g(a)\big) \perp\!\!\!\perp \mathbb{1}\{A = g(a)\} \mid A \in S_g(a) \qquad \forall\, a \in \mathcal{A}$$

We also consider

$$Y\big(g(a)\big) \perp\!\!\!\perp A \mid A \in S_g(a) \qquad \forall\, a \in \mathcal{A}$$

This distinction becomes important in the longitudinal setting.

Example

Let $A \in \{1, 2, 3\}$ and

$$A^+(g) = g(A) = \begin{cases} 2 & \text{if } A = 1, \\ 3 & \text{if } A = 2, \\ 1 & \text{if } A = 3. \end{cases}$$

Then the new condition is

$$Y\big(g(a)\big) \perp\!\!\!\perp \mathbb{1}\{A = g(a)\} \mid A \in \underbrace{\{g(a)\} \cup \{a\}}_{=S_g(a)} \qquad \forall\, a \in \mathcal{A}$$

In particular, for $a = 1$, the condition is

$$Y\big(2\big) \perp\!\!\!\perp A \mid A \in \underbrace{\{1, 2\}}_{=S_g(1)}$$

Sufficient for identification

$$\begin{aligned}
E\big[Y\big(g(A)\big)\big] &= \sum_{a\in\mathcal{A}} E\big[Y\big(g[a]\big) \mid A = a\big]\, P(A = a) \\
&= \sum_{a\in\mathcal{A}} E\big[Y\big(g[a]\big) \mid A = a,\ A \in S_g(a)\big]\, P(A = a) \\
&= \sum_{a\in\mathcal{A}} E\big[Y\big(g[a]\big) \mid A = g[a],\ A \in S_g(a)\big]\, P(A = a) \\
&= \sum_{a\in\mathcal{A}} E\big[Y\big(g[a]\big) \mid A = g[a]\big]\, P(A = a) \\
&= \sum_{a\in\mathcal{A}} E\big[Y \mid A = g[a]\big]\, P(A = a)
\end{aligned}$$

We only need exchangeability to hold conditional on $A \in \{g(a)\} \cup \{a\}$.

Reduces to well-known versions

Under a static regime

$$A^+(g) = g(A) = \tilde{a} \qquad \text{for } \tilde{a} \in \mathcal{A},$$

we have

$$S_g(a) = \{g(a)\} \cup \{a' \in \mathcal{A} : g(a') = g(a)\} = \mathcal{A} \qquad \forall\, a \in \mathcal{A},$$

so the conditions reduces to

$$Y(\tilde{a}) \perp\!\!\!\perp \mathbb{1}\{A = \tilde{a}\},$$

and

$$Y(\tilde{a}) \perp\!\!\!\perp A.$$

Holds trivially under no intervention

Under the no-intervention regime

$$A^{+}(g) = g(A) = A,$$

we have

$$S_g(a) = \{a\} \qquad \forall\, a \in \mathcal{A},$$

so the conditions becomes

$$Y(a) \perp\!\!\!\perp \mathbb{1}\{A = a\} \mid A = a \qquad \forall\, a \in \mathcal{A},$$

and

$$Y(a) \perp\!\!\!\perp A \mid A = a \qquad \forall\, a \in \mathcal{A},$$

which holds trivially.

Generalizes the MCM condition

The new condition includes the Minimal Counterfactual Model (MCM) condition for static regimes (Robins & Richardson, 2010) as a special case:

$$Y(a) \perp\!\!\!\perp \mathbb{1}\{A = a\} \qquad \forall\, a \in \mathcal{A}, \tag{1}$$

However, we show that this condition is not sufficient for identification of NTV regimes, which clarifies confusion about sufficient conditions in the existing literature.

Relation to other conditions

The following condition is sometimes stated

$$Y(a) \perp\!\!\!\perp A \qquad \forall\, a \in \mathcal{A},$$

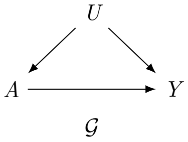
because it implies the MCM condition.
(E.g., Sarvet & Stensrud, 2026)

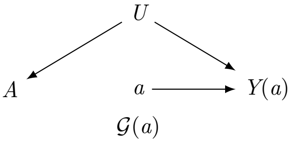
Our new identification result confirms that this condition is sufficient.

Graphical evaluation

FFRCISTG model wrt. $\mathcal{G}$

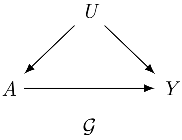
Assume an FFRCISTG model wrt. $\mathcal{G}$

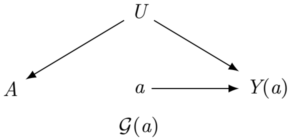




$\mathcal{G}(a)$ suggests that $E[\,Y(g)]$ is not identified.

Regime-specific information





Suppose that a subject-matter expert is convinced that

$$U \perp\!\!\!\perp Y(g[a]) \mid A \in S_g(a) \qquad \forall\, a \in \mathcal{A}.$$

We consider a concrete clinical example with antibiotic treatments in the paper.

All we need is $A \in \{1, 2, 3\}$.

Limitation: this information cannot be encoded in $\mathcal{G}$ or $\mathcal{G}(a)$.

Alternative: Regime-specific SWIG model

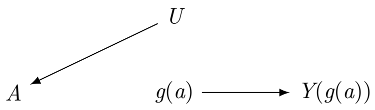
Assume that the joint distribution of $(A, U, Y(g(a)))$ conditional on $\{A \in S_g(a)\}$ factorizes according to a regime-specific SWIG $\mathcal{G}_g(a)$.

Informally, regime-specific SWIGs

- look like normal SWIGs;
- are not constructed from an underlying DAG $\mathcal{G}$;
- can therefore encode regime-specific information.

Example: Regime-specific SWIG model

$A \in S_g(a)$:



$\mathcal{G}_g(a)$

$$Y(g(a)) \perp\!\!\!\perp_{\mathcal{G}_g(a)} A \qquad \forall\, a \in \mathcal{A} \quad ✓$$

$$\Rightarrow \quad Y\big(g(a)\big) \perp\!\!\!\perp \mathbb{1}\{A = g(a)\} \mid A \in S_g(a) \qquad \forall\, a \in \mathcal{A}$$

$$\Rightarrow \quad E[\,Y(g)] = \sum_{a \in \mathcal{A}} E\big[\,Y \mid A = g[a]\big]\; P(A = a)$$

Time-varying setting

Generalization

A regime g is a current NTV regime if

$$A_k^+(g) = g_k\big(\bar{L}_k(g),\, \bar{A}_{k-1}^+(g),\, A_k(g)\big) \quad \text{a.s.} \quad \forall\, k \leq K,$$

and a general regime if

$$A_k^+(g) = g_k\big(\bar{L}_k(g),\, \bar{A}_{k-1}^+(g),\, \bar{A}_k(g)\big) \quad \text{a.s.} \quad \forall\, k \leq K,$$

A generalization of the following is sufficient for current NTV regimes:

$$Y\big(g(a)\big) \perp\!\!\!\perp \mathbb{1}\{A = g(a)\} \mid A \in S_g(a) \qquad \forall\, a \in \mathcal{A}.$$

A generalization of the following is sufficient for general regimes:

$$Y\big(g(a)\big) \perp\!\!\!\perp A \mid A \in S_g(a) \qquad \forall\, a \in \mathcal{A}.$$

Forthcoming work by Stoltenberg, Richardson, Robins & Stensrud (2026+)

A common class of regimes

Most (all?) regimes in the literature are current NTV regimes:

- Static regimes
- Conventional dynamic regimes
- Popular NTV regimes (Sarvet and Stensrud, 2025):
  - Point treatment regimes

    Including superoptimal regimes (Stensrud et al. 2024, Laurendeau et al. 2025)
  - Relative shift interventions

    (Robins et al. 2004, Robins 2008, Danaei et al. 2013, Lajous et al. 2013, Garcia-Aymerich et al. 2014)
  - Threshold interventions

    (Taubman et al. 2009, Cole et al. 2013, Danaei et al. 2013, Edwards et al. 2014, Garcia-Aymerich et al. 2014)
- Add-on regimes (Stoltenberg et al. 2025)

Summary

The new exchangeability conditions are

- General: cover a large class of treatment regimes
- Unified: reduce to well-known conditions in standard settings
- Minimal: generalizes the MCM condition (Robins & Richardson, 2010)

The new regime-specific SWIG model can incorporate regime-specific information.